



SPHALERONS FAR FROM EQUILIBRIUM AND ASSOCIATED PHENOMENA

MARK MACE
STONY BROOK UNIVERSITY &
BROOKHAVEN NATIONAL LAB
BASED ON: MM, S. SCHLICHTING, R.
VENUGOPALAN ARXIV:1601.07342 [HEP-PH]

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OUTLINE

- ▶ Motivation: Chiral Magnetic Effect
- ▶ Real-Time Lattice Gauge Theory
- ▶ Sphalerons: In Equilibrium
- ▶ Out of Equilibrium
 - ▶ Scales
 - ▶ Sphalerons
- ▶ Future Outlook

MOTIVATION: THE CHIRAL MAGNETIC EFFECT

- ▶ Axial anomaly \rightarrow non conservation of axial current
- ▶ Axial Current: $j_5^\mu = j_R^\mu - j_L^\mu$
- ▶ Quantum effect

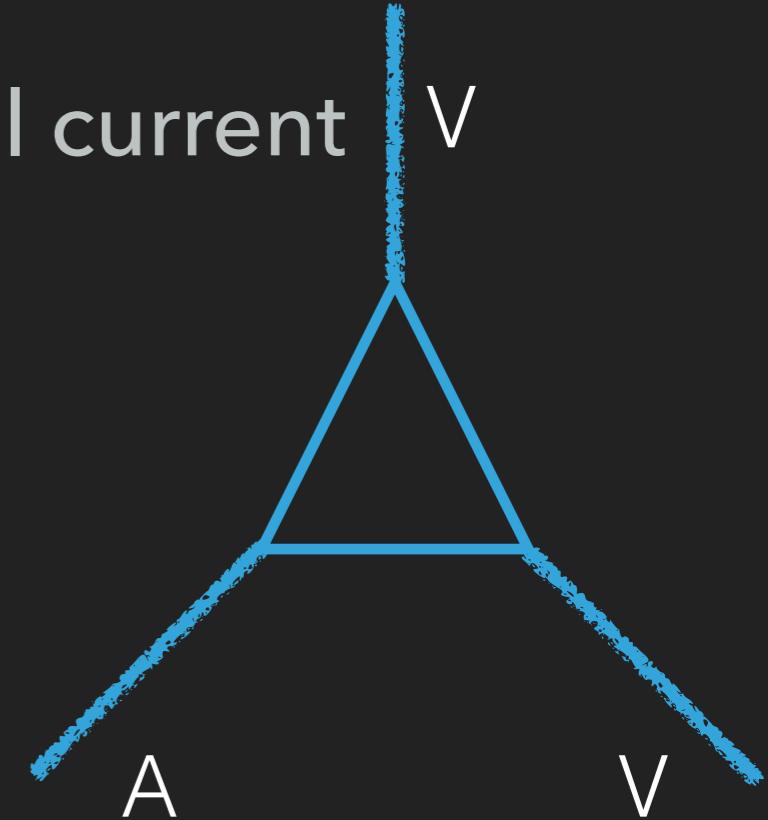
$$\partial_\mu j_{5,f}^\mu = 2m_f \bar{q} \gamma_5 q - \frac{g^2}{16\pi^2} F_{\mu\nu}^a \tilde{F}_a^{\mu\nu}$$



Axial charge

$$J_5^0 = \int d^3x j_5^0(x)$$

Nonzero from topological transitions, field strength fluctuations, ...



- ▶ One can define a Chern-Simons current

$$K^\mu = \frac{g^2}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} \left(F_{\nu\rho}^a A_\sigma^a - \frac{g}{3} f_{abc} A_\nu^a A_\rho^b A_\sigma^c \right)$$

- ▶ Recast anomaly equation as $\partial_\mu j_5^\mu = -2N_f \partial_\mu K^\mu$

→ $\frac{g^2}{32\pi^2} F_{\mu\nu}^a \tilde{F}_a^{\mu\nu} = \partial_\mu K^\mu$

- ▶ Define Chern-Simons number

$$N_{CS}(t) = \int d^3x K^0(x, t)$$

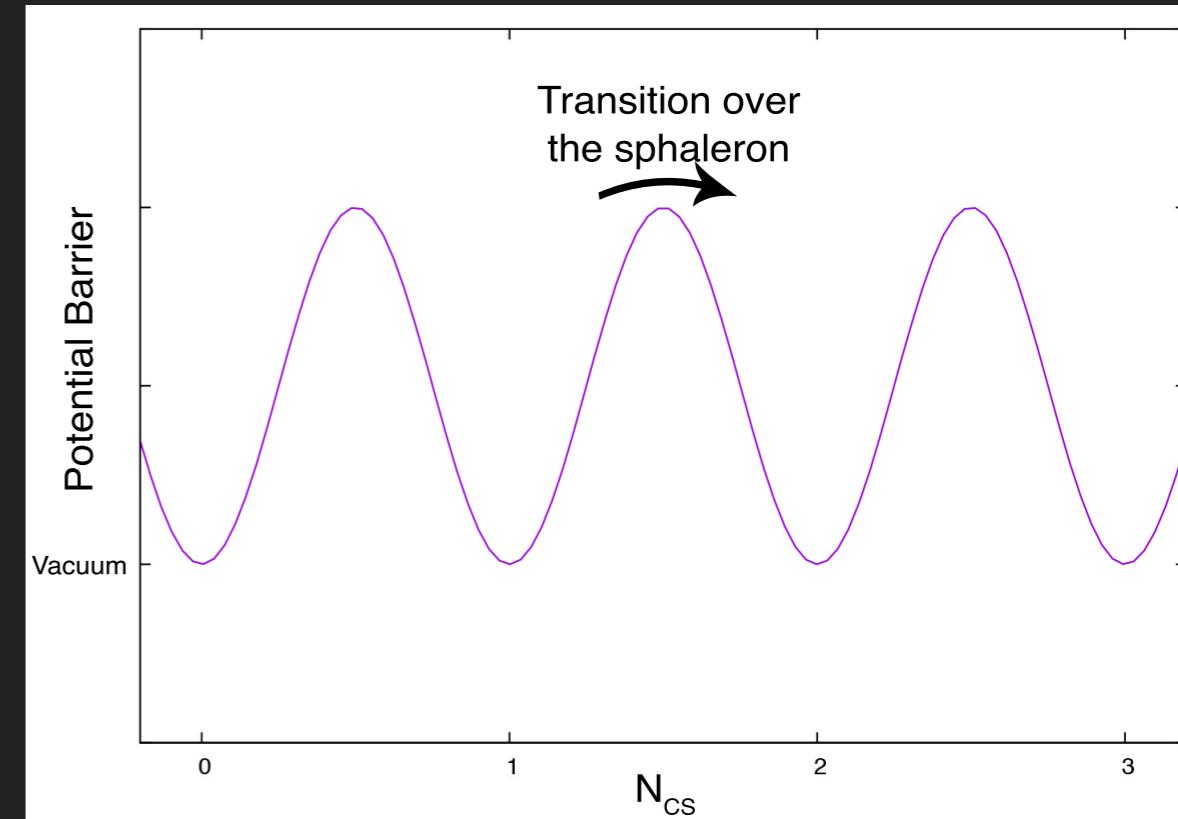
- ▶ Change in N_{cs} (ex. via topological transitions) can give global axial charge imbalance

$$\Delta J_5^0 = -2N_f \int dt \partial_0 K^0 = -2N_f \Delta N_{CS}$$

$$\Delta J_5^0 = N_R - N_L$$

CHERN-SIMONS NUMBER VIA SPHALERON PROCESS

- ▶ We are interested in the real time topological transitions, aka sphaleron transitions
 - ▶ Transition over the barrier denoted by integral change in N_{cs}
 - ▶ These are called sphalerons from Greek for “ready to fall”
 - ▶ Static, unstable saddle-point solution
 - ▶ Dominant at high temperatures, like early universe or heavy ion collisions



CHIRAL MAGNETIC EFFECT IN TWO LINES

- ▶ Anomaly creates local P-odd bubbles (non-zero axial charge density)

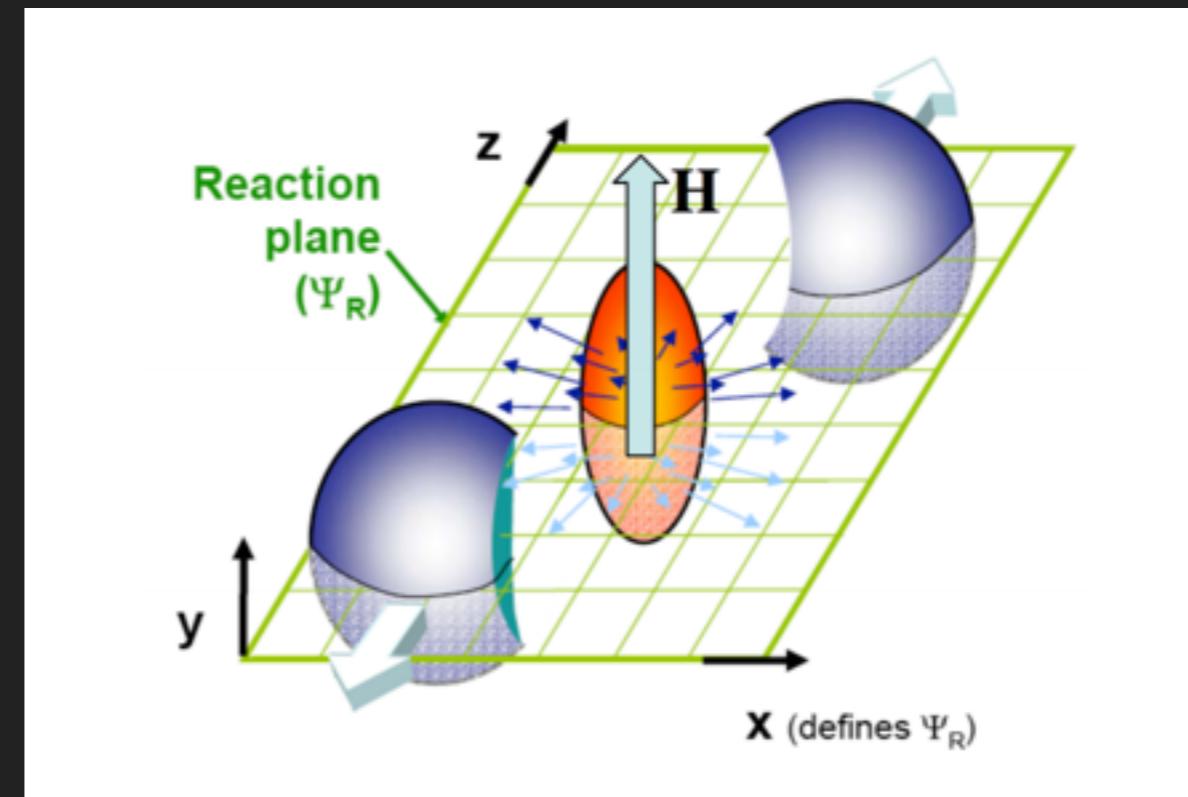
$$\Delta Q_5^0 \sim \int d^4x F_{\mu\nu}^a \tilde{F}_a^{\mu\nu} \sim \int d^4x \vec{E}^a \cdot \vec{B}^a$$

- ▶ External U(1) magnetic field creates a vector current

$$\mu_5 = \frac{1}{2}(\mu_R - \mu_L)$$

$$\mathbf{J}_V = \frac{g^2}{2\pi^2} \mu_5 \mathbf{B}$$

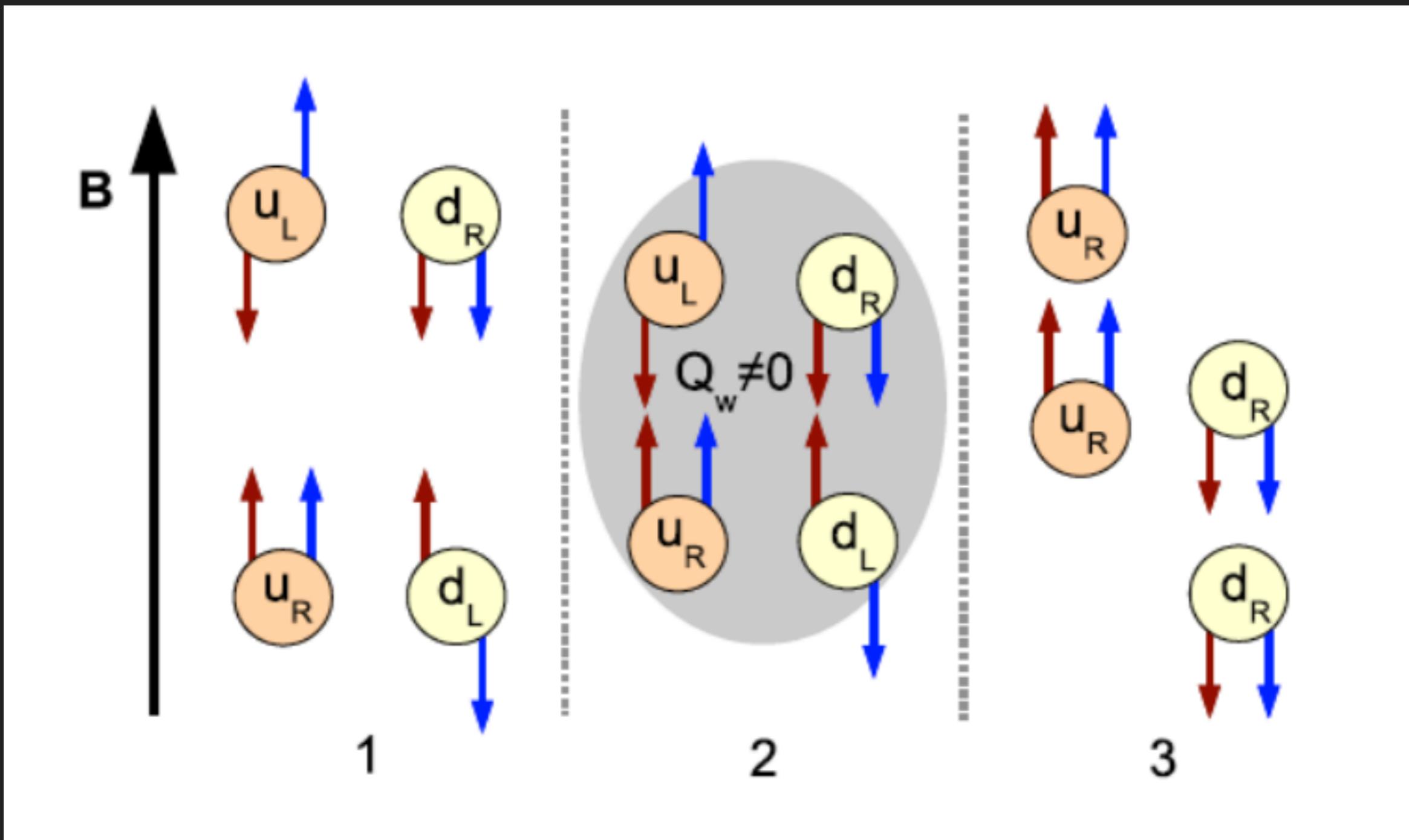
IMAGE FROM ARXIV
1312.3348



KHARZEEV, MCLERRAN, AND WARRINGA , NUCL. PHYS. A803, 227 (2008), 0711.0950.

FUKUSHIMA, KHARZEEV, AND WARRINGA PHYS.REV. D78, 074033 (2008), 0808.3382.

CME QUALITATIVELY



Red arrows- momentum; blue arrows-spin

MAGNETIC FIELD IN HEAVY ION COLLISIONS

- ▶ Off-central collisions create strong B-field

$$▶ eB \approx (m_\pi)^2 \sim 10^{18} \text{G}$$

- ▶ However it is very short lived

$$▶ \sim 0.1\text{-}1 \text{ fm/c}$$

- ▶ Poses practical challenge

Theoretical model calculation

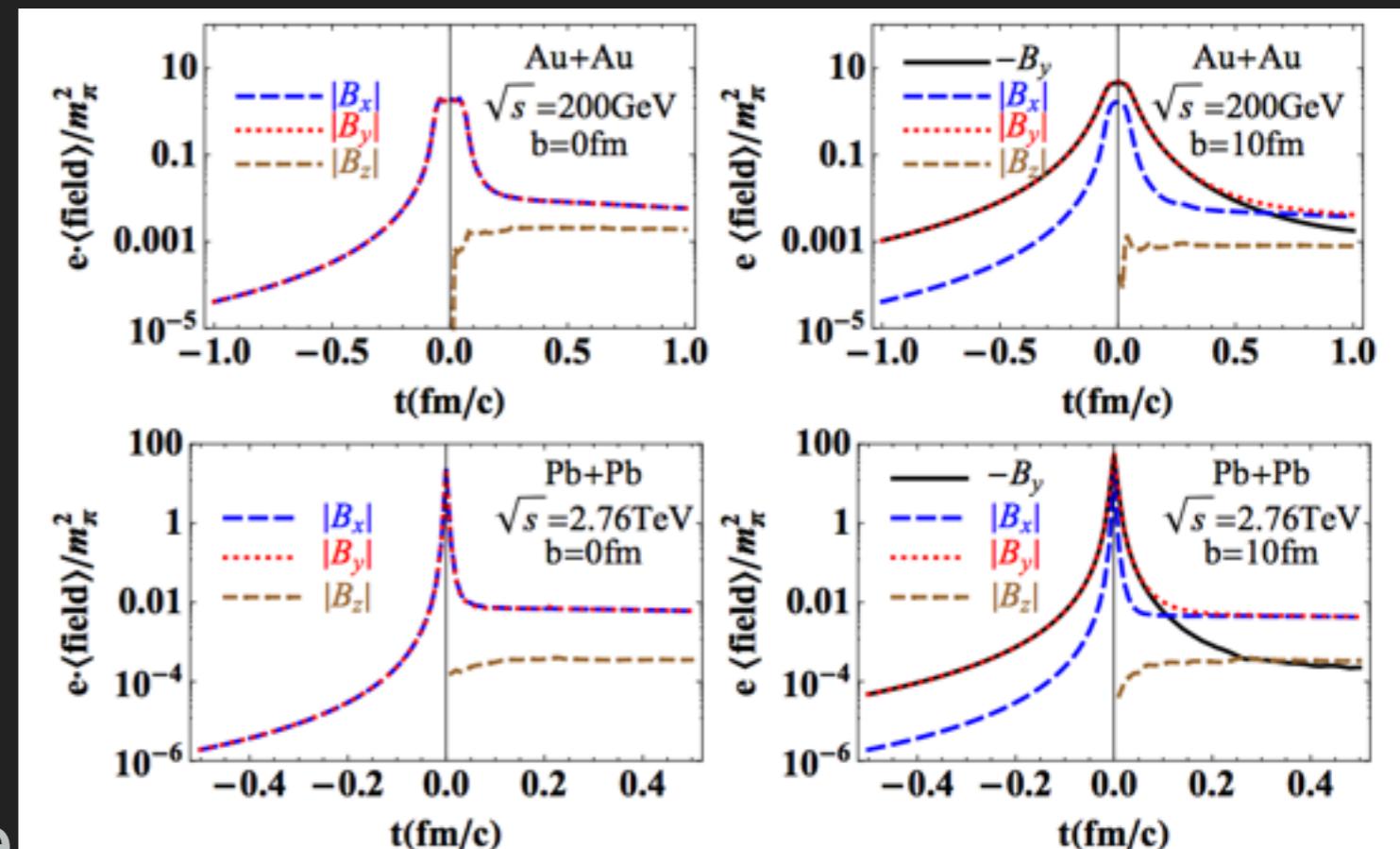


IMAGE FROM; W.-T. DENG AND X.-G. HUANG, PHYS. REV. C85, 044907 (2012), 1201.5108.

CHIRAL MAGNETIC EFFECT IN REALITY

- ▶ Heavy ion collision experiments
 - ▶ STAR, ALICE studied CME (and other anomalous transport phenomena), no conclusive results yet
 - ▶ Alternative explanations exist
 - ▶ Local charge conservation
 - ▶ Viscous hydrodynamics
- ▶ Recent discovery in condensed matter systems

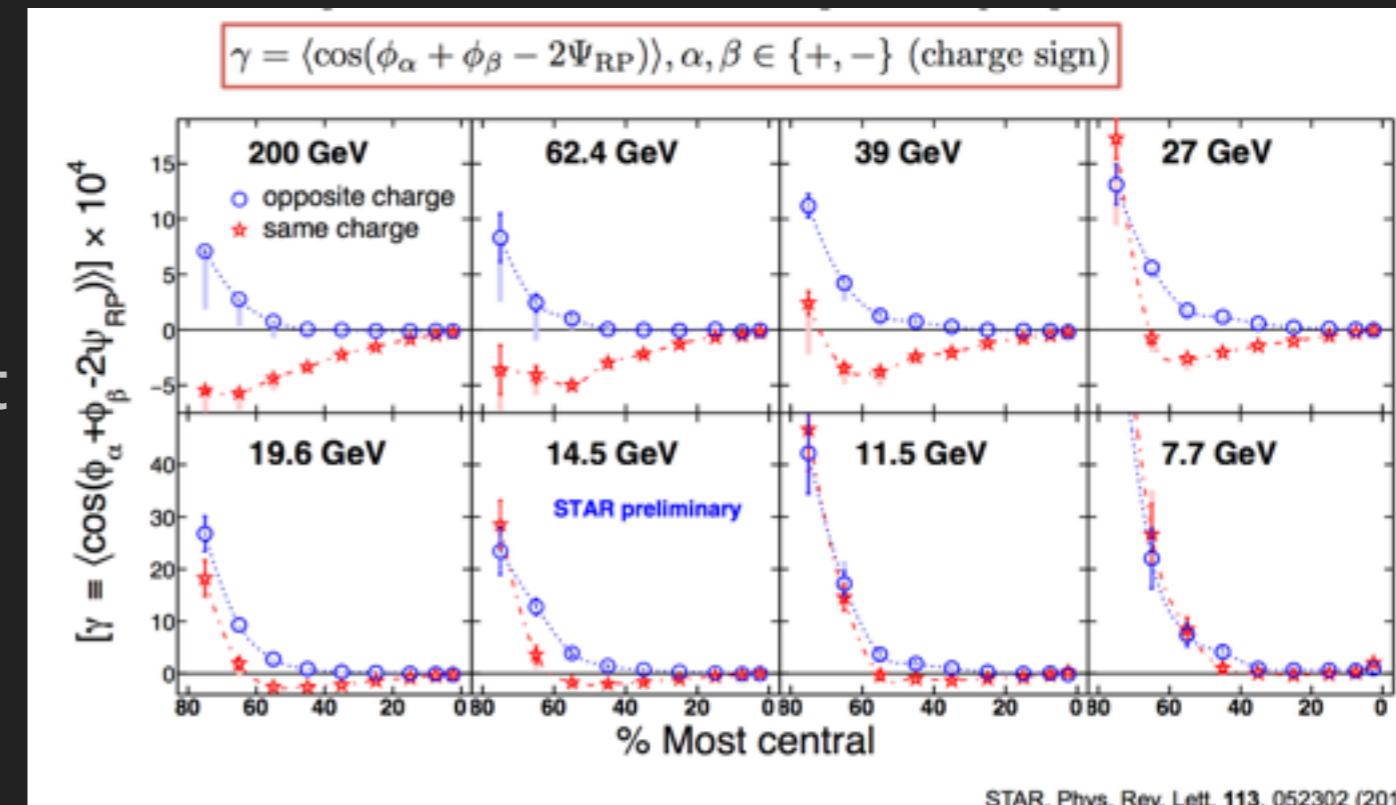
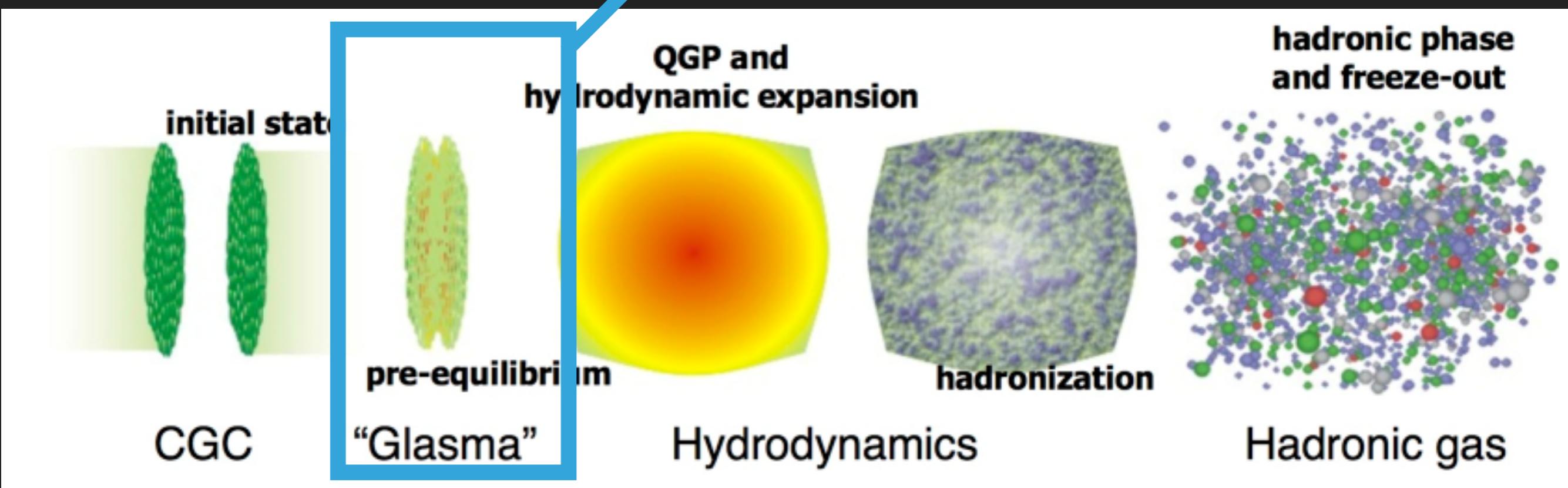


Image from H. Ke's RHIC/AGS 2015 users meeting talk

OUR MOTIVATION

- ▶ Effects of magnetic field largest during early times of HIC
- ▶ System is very far from equilibrium
- ▶ Important to understand early time axial charge generation from first principles to achieve a truly ab initio calculation of the CME in heavy ion collisions
- ▶ At what rate do topological transitions occur in a non-Abelian plasma out of equilibrium?

TIMELINE OF A HIGH ENERGY HEAVY ION COLLISION



GLASMA: NON-EQUILIBRIUM, GLUON DOMINATED MATTER

PHYSICAL SETTING

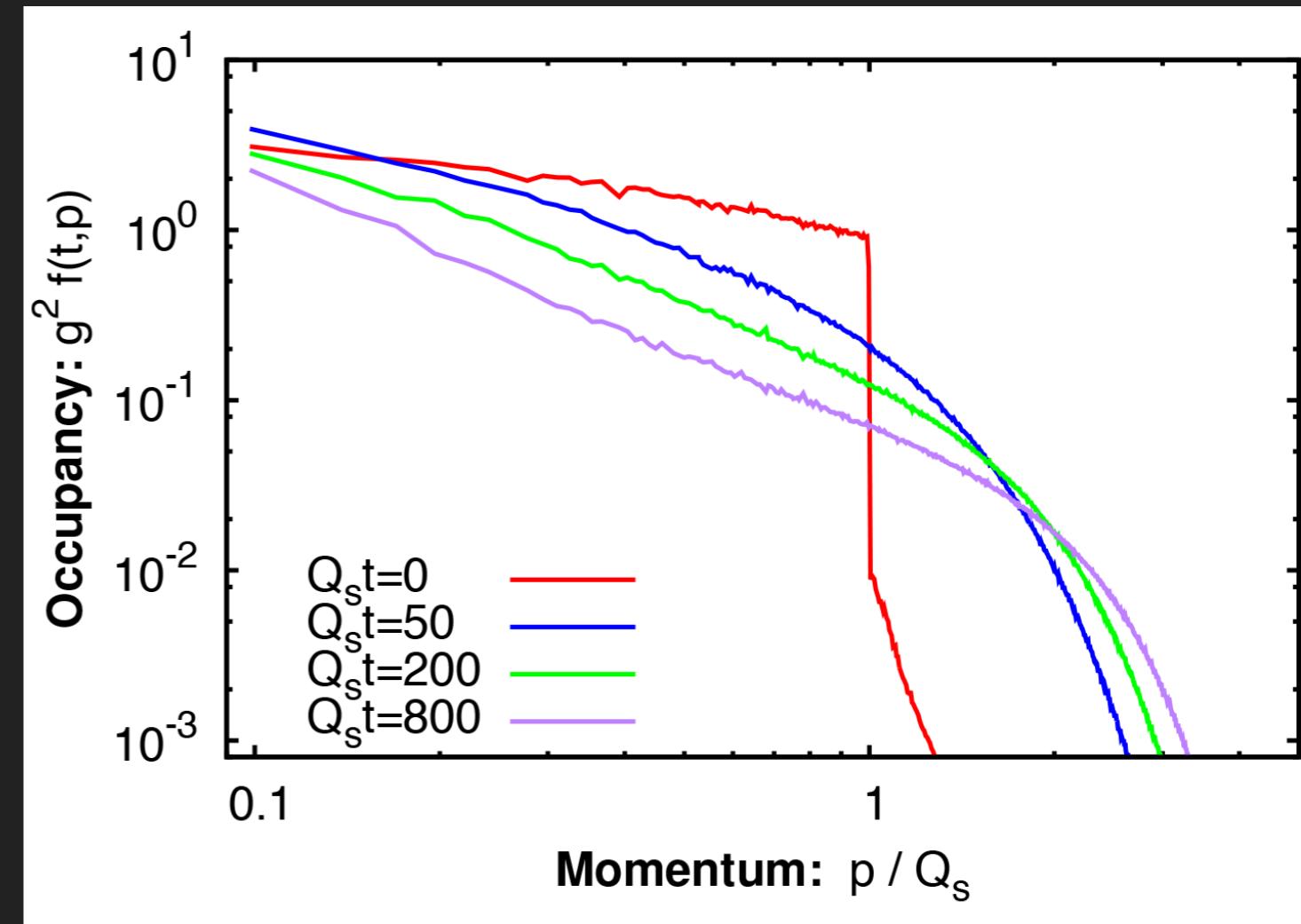
- ▶ Glasma:
 - ▶ Very far from equilibrium
 - ▶ Weak coupling $\alpha_S \ll 1$
 - ▶ Highly over occupied
 - ▶ Large phase-space density of gluon, $f \sim \frac{1}{\alpha_S}$
 - ▶ Non-perturbative
 - ▶ Amenable to classical-statistical description

OUR TOOLBOX

- ▶ Consider only the highly occupied gluon sector of QCD
- ▶ Fermions not considered in first study
- ▶ Study classical Yang-Mills (CGC)
- ▶ Consider SU(2) as first study
- ▶ Non-expanding (fixed) box as a first calculation
- ▶ Work in classical-statistical framework on the lattice
- ▶ Important: Must sample over ensemble of stochastic initial conditions

NON-EQUILIBRIUM INITIAL CONDITIONS

- ▶ Quasi-particle picture
- ▶ Superposition of transversely polarized gluon states
- ▶ Initially, only one scale, Q_s (CGC)



$$A_\mu^a(t_0, x) = \sum \int \frac{d^3 k}{(2\pi)^2} \frac{1}{2k} \sqrt{f(t_0, k)} [c_k^a \xi_\mu^\lambda(k) e^{ikx} + c.c.]$$

$$E_\mu^a(t_0, x) = \sum \int \frac{d^3 k}{(2\pi)^2} \frac{1}{2k} \sqrt{f(t_0, k)} [c_k^a \dot{\xi}_\mu^\lambda(k) e^{ikx} + c.c.]$$

CLASSICAL YANG-MILLS DYNAMICS

- ▶ Recast Hamiltonian in terms of lattice variables

- ▶ Gauge fields $A \rightarrow U$

$$\frac{\delta U_\mu(x)}{\delta A_\nu^a(y)} = -iga\tau^a U_\mu(x) \delta_\mu^\nu \frac{\delta_{x,y}}{a^3}$$

- ▶ Electric Fields $E \rightarrow E$

- ▶ Use Kogut-Susskind YM Hamiltonian in terms of products of gauge links (plaquettes) and conj. E

$$H = \frac{a^3}{2} \sum_{j,x} E_j^a(x) E_j^a(x) + \frac{2}{g^2 a} \sum_{\square} \text{ReTr} [I - U_\square]$$

$$\partial_t E_a^\mu(x) = -\frac{\delta H}{\delta A_\mu^a(x)}$$

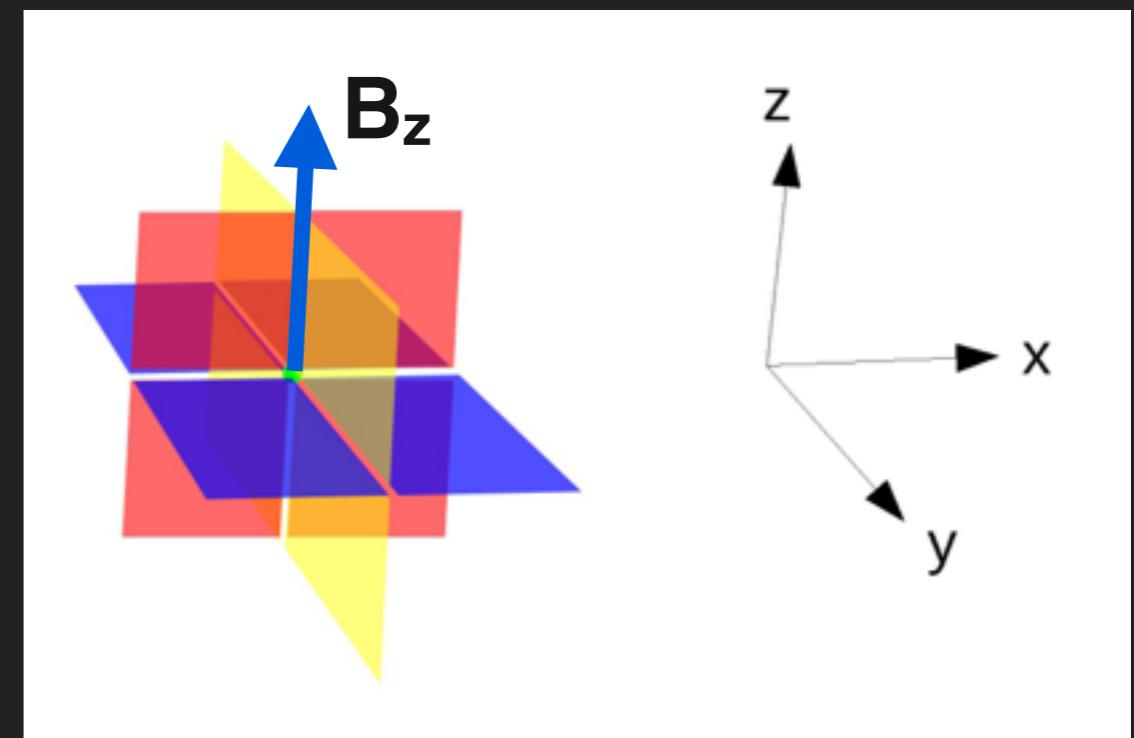
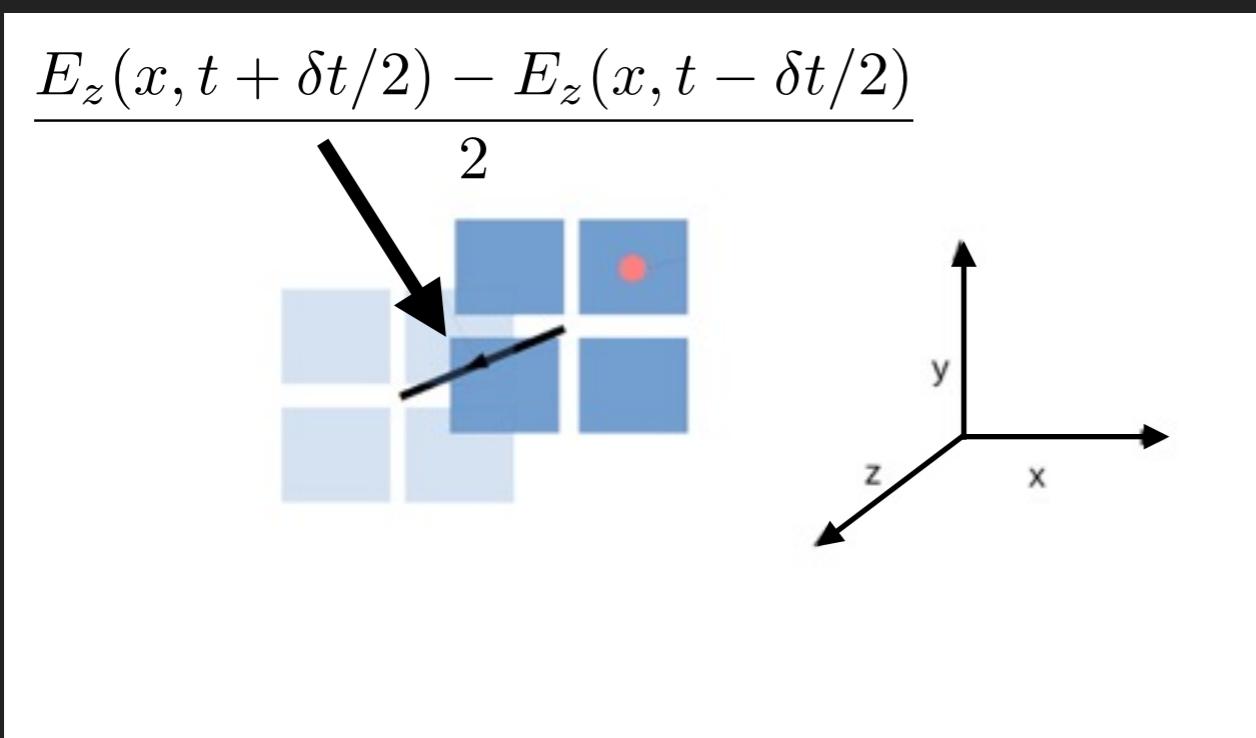
Eqns
of motion

$$\partial_t U_\mu(x) = -iga \tau^a \frac{\delta H}{\delta E_a^\mu(x)} U_\mu(x)$$

TOPOLOGY ON THE LATTICE

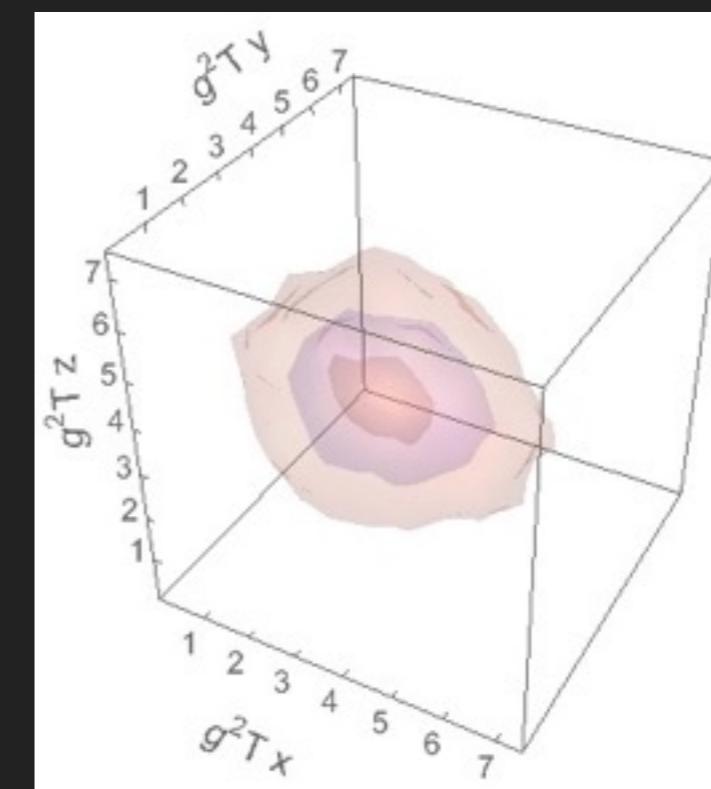
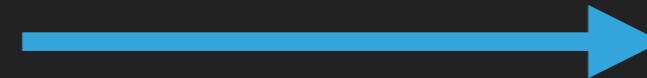
- ▶ $\frac{dN_{CS}(t)}{dt} = \int d^3x \frac{g^2}{8\pi^2} E_i^a B_i^a$
- ▶ On the lattice,

$$\frac{dN_{CS}}{dt} \simeq \frac{g^2}{8\pi^2} \sum_{\mathbf{x} \in lattice, i} \frac{(E_i^a(\mathbf{x}, t + \delta t/2) + E_i^a(\mathbf{x}, t - \delta t/2))}{2} 2 \sum_{8\square} \text{tr}\left(\frac{i\tau^a U_\square(\mathbf{x})}{4}\right)$$



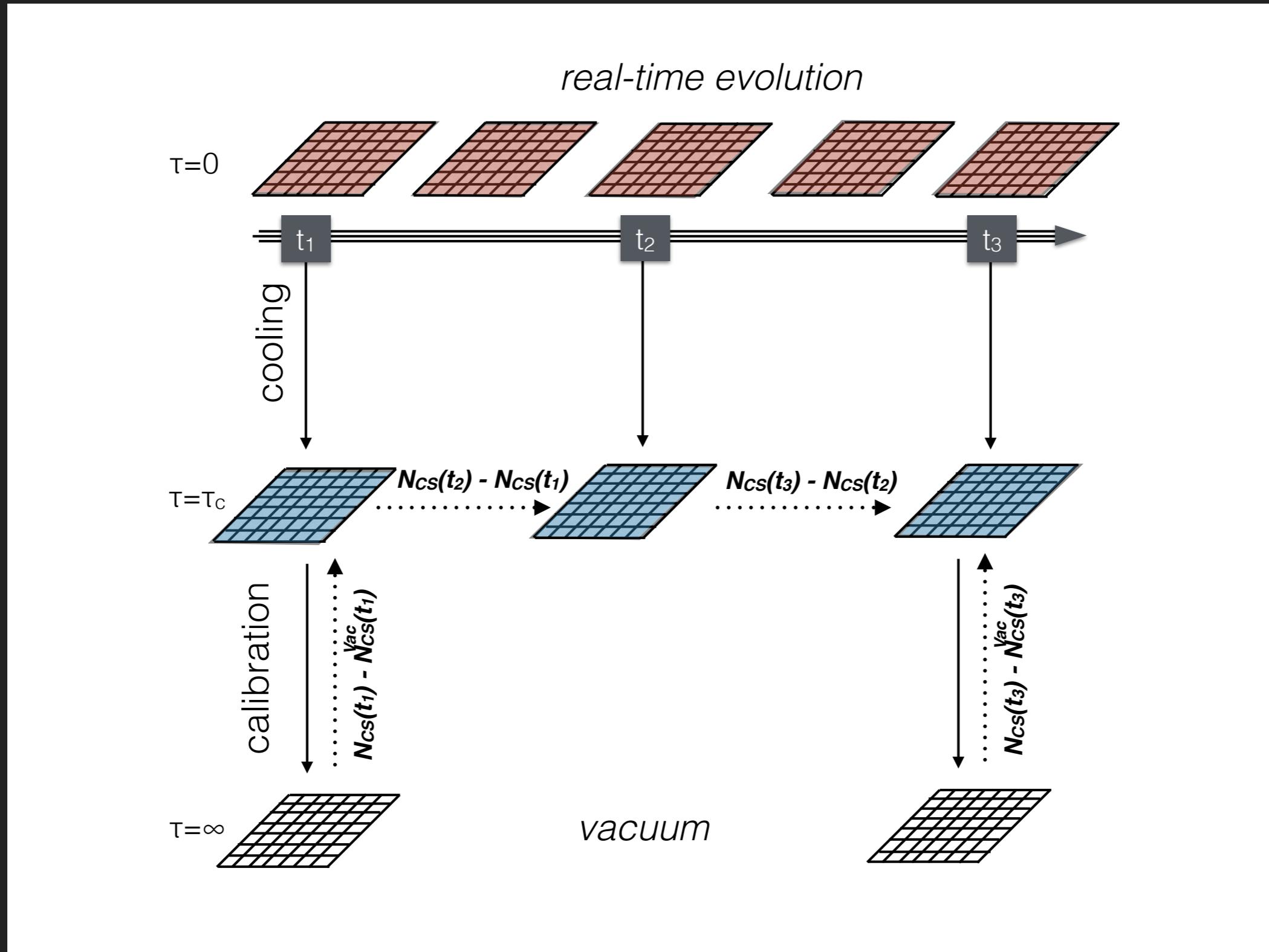
TOPOLOGY ON THE LATTICE

- ▶ Integrate over time to get ΔN_{CS}
- ▶ Defining topology on lattice challenging
 - ▶ Highly susceptible to UV noise
 - ▶ No local operator definition of $\vec{E}^a \cdot \vec{B}^a$ on the lattice that is a total derivative
 - ▶ Need smoother configurations – COOLING



COOLING: A CARTOON

$$\frac{\partial A_i^a(x)}{\partial \tau} = -\frac{\partial H}{\partial A_i^a(x)}$$



MEASURING SPHALERONS: THERMAL EQUILIBRIUM

- ▶ First case: Thermal equilibrium $f \sim T/p$
 - ▶ Historically for baryogenesis calculations
 - ▶ Regime where classical Yang-Mills (or +Higgs) relevant
 - ▶ Considered sphalerons before and after EW phase transition
 - ▶ Results well known
 - ▶ Ambjorn & Krasnitz, Guy Moore (& various collaborators)

POWER COUNTING THE RATE

- ▶ (Equilibrium) sphaleron transition rate

$$\Gamma = \lim_{t \rightarrow \infty} \frac{\langle (N_{CS}(t) - N_{CS}(0))^2 \rangle}{Vt}$$

- ▶ Dimensionally $\Gamma \sim T^4$
- ▶ What physics dominates at high temperatures?
 - ▶ In equilibrium at weak coupling, hierarchy of scales

Hard momentum

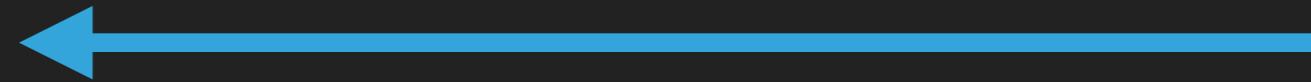
$$\Lambda \sim T$$

Electric

$$m_D \sim gT$$

Magnetic

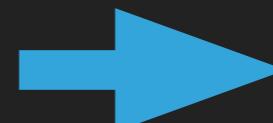
$$\Lambda_s \sim g^2 T$$



Strength

WHERE DOES THE RATE COME FROM?

- ▶ Sphaleron transitions dominated by long wavelength modes – issues with the cutoff

- ▶ Spatial and time modes $\Lambda_s \sim g^2 T$  $\Gamma_{eq} \sim \alpha^4 T^4$

- ▶ Alternatively, if Landau dampening controls temporal scale

$$t \sim 1/(g^4 T) \rightarrow \Gamma_{eq} \sim \alpha^5 T^4$$

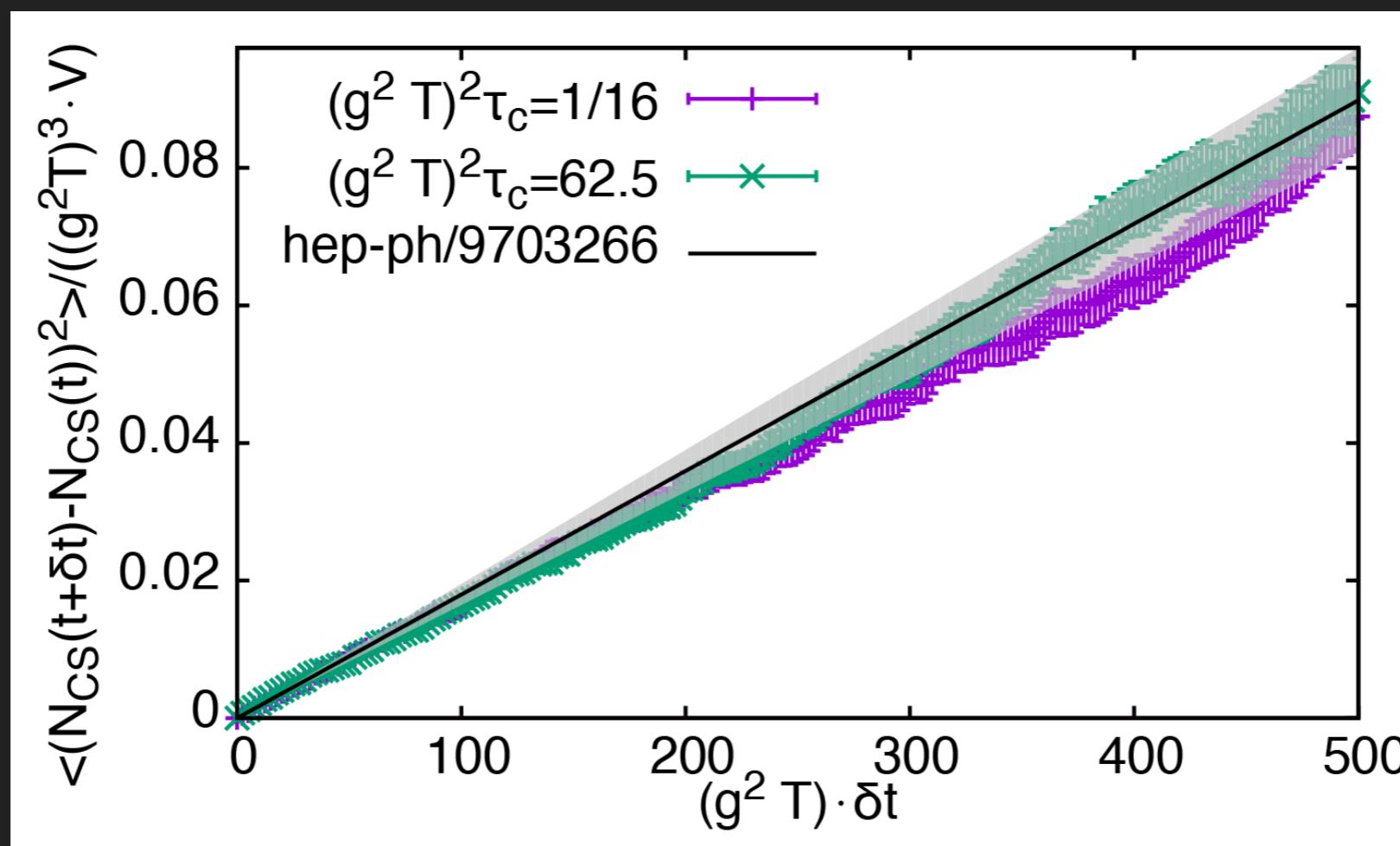
- ▶ Need effective theory on lattice for coupling of hard modes to soft modes -> Necessary for UV sensitivity of equilibrium sphaleron transition rate

- ▶ Latest SU(2) result: $\Gamma_{eq} \sim 25\alpha^5 T^4$ (Moore et al)

KUZMIN, RUBAKOV, AND SHAPOSHNIKOV, PHYS. LETT. B155, 36 (1985). ARNOLD AND MCLERRAN, PHYS. REV. D37, 1020 (1988). ARNOLD, SON, AND YAFFE, PHYS. REV. D55, 6264 (1997), BODEKER PHYS. LETT. B426 (1998) 351-360, G. D. MOORE, C.-R. HU, AND B. MULLER, PHYS. REV. D58, 045001 (1998) G. D. MOORE AND M. TASSLER, JHEP 02, 105 (2011).

THERMAL 2016

- ▶ Study correlations in $(\Delta N_{CS})^2$ on lattice
- ▶ Find agreement with established thermal equilibrium studies (not including effective theory)



CONNECTING THERMAL TO OUT OF EQUILIBRIUM

- ▶ In thermal equilibrium, a weakly coupled non-Abelian plasma is described by these three scales, hard momentum scale, electric and magnetic scales, separated in powers of the coupling -> Can write an effective theory
- ▶ In the glasma, all start out parametrically of order Q_s
 - ▶ Not a problem in our approach
 - ▶ Can observe separation of scales as a function of time
 - ▶ Can parameterize the transition rate in terms of one or multiple physical scales

HARD MOMENTUM SCALE

- ▶ Typical momentum scale for hard modes
- ▶ Increases with time (towards thermalization)
- ▶ In equilibrium is $\Lambda \sim T$
- ▶ In quasi particle picture $\Lambda^2(t) \simeq \frac{\int d^3\mathbf{p} p^2 \omega_p f(\mathbf{p}, t)}{\int d^3\mathbf{p} \omega_p f(\mathbf{p}, t)}$
- ▶ Gauge invariant lattice definition possible
- ▶ Kinetic theory $\Lambda \sim Q_s (Q_s t)^{1/7}$

ELECTRIC SCALE

- ▶ Debye mass
- ▶ In equilibrium is $m_D \sim gT$
- ▶ Characterizes static (chromo-)electric field screening at large distances
- ▶ Defined perturbatively

$$m_D^2 \simeq 4N_c g^2 \int \frac{d^3 p}{2\pi^3} \frac{f(p)}{p}$$

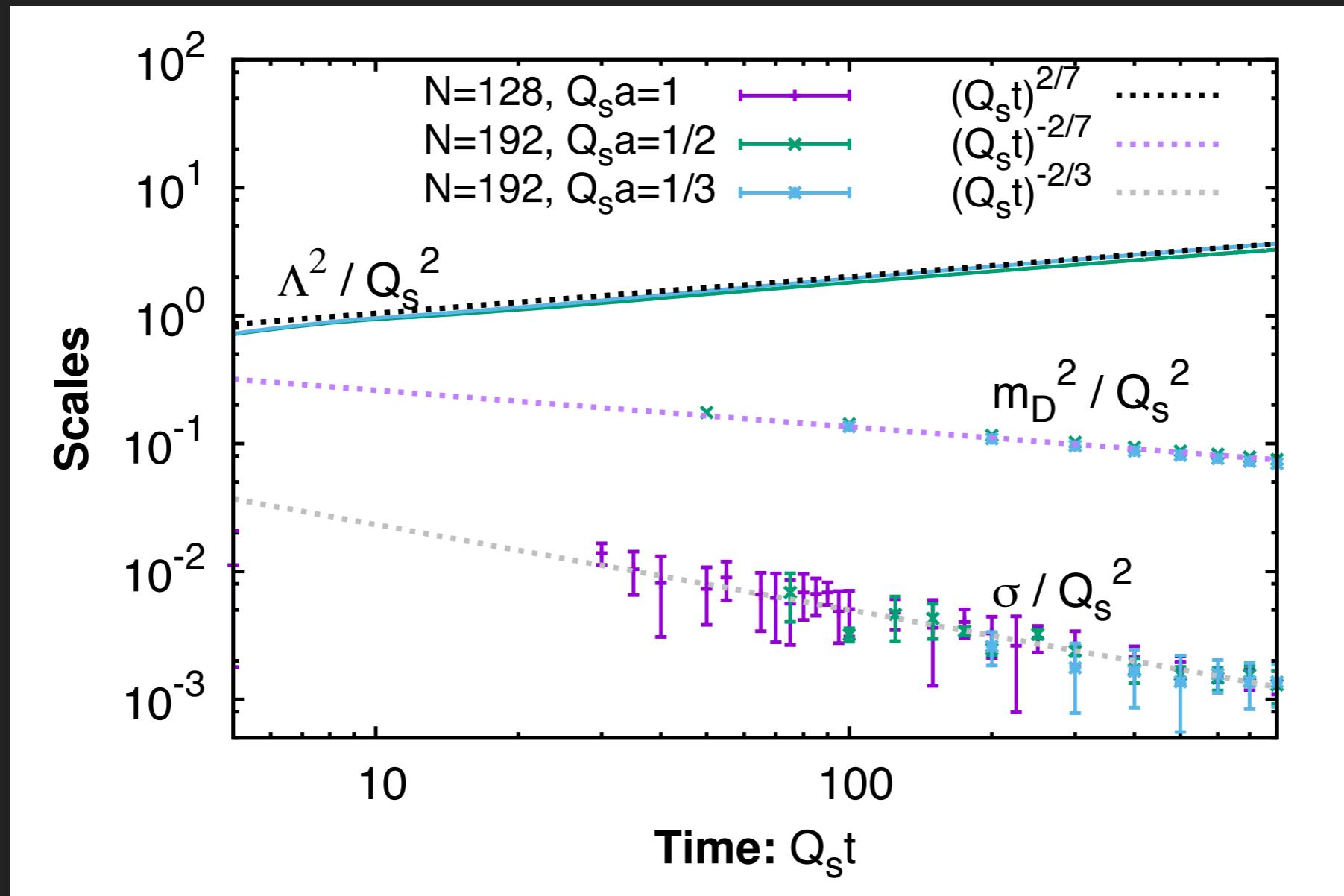
- ▶ From kinetic theory

$$m_D \sim Q_s (Q_s t)^{-1/7}$$

MAGNETIC SCALE

- ▶ In equilibrium $\Lambda_s \sim g^2 T$
- ▶ First systematic real-time lattice calculation
- ▶ Calculate string tension as a proxy for magnetic scale
 - ▶ Spatial Wilson loop simplest gauge invariant object to study magnetic screening
 - ▶ Area law $\langle W(t, A) \rangle \sim P e^{ig \oint_C A \cdot d\ell} \sim e^{-\sigma A}$
 - ▶ Get string tension $\sigma(t) = -\frac{\partial \ln W(t, A)}{\partial A}$
 - ▶ (2+1)D case done already (Dumitru, Lappi, and Nara)
 - ▶ Kinetic theory $\sqrt{\sigma} \sim Q_s (Q_s t)^{-3/7}$

NON-EQUILIBRIUM SCALE SEPARATION



- Kinetic theory prediction scorecard: 2 out of 3

RETURN OF THE SPHALERON

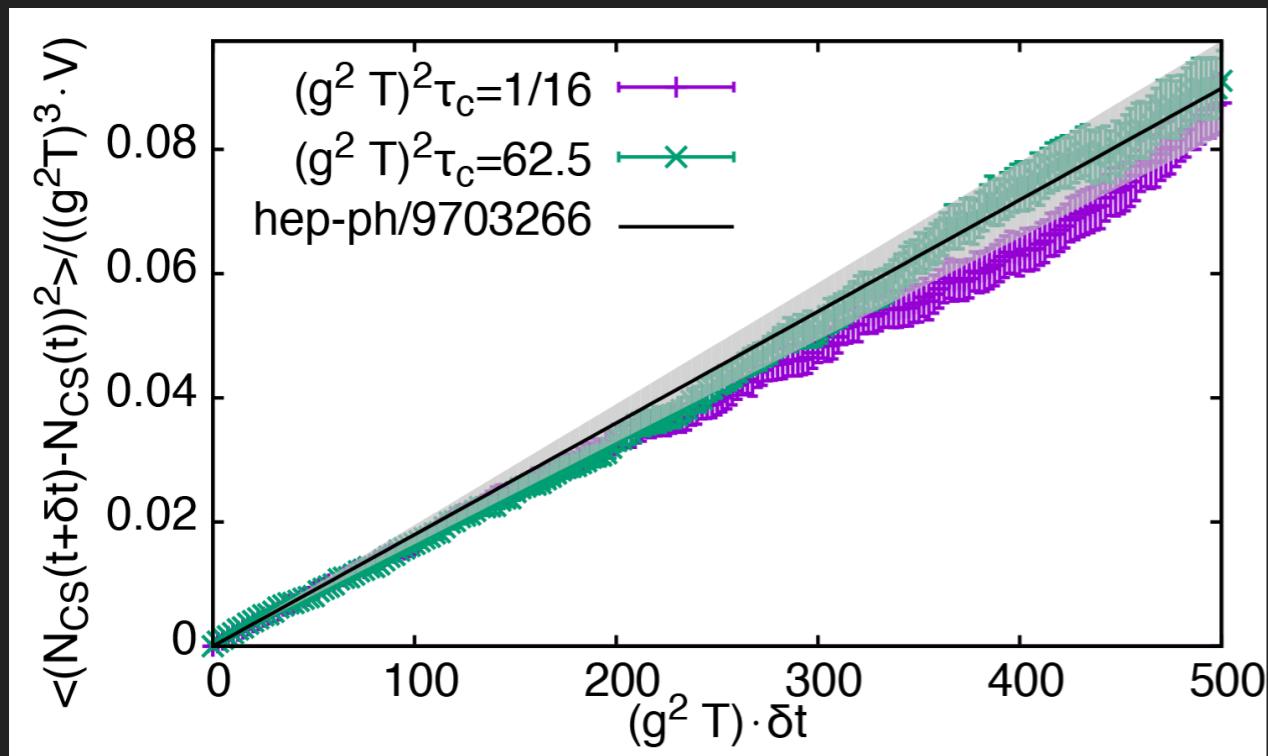
- ▶ Now we measure sphalerons with our out of equilibrium configurations
- ▶ Generate configurations over wide range of lattice size and spacings, cooling depth
- ▶ Dimensionally we expect a rate:

$$\Gamma = \kappa Q_s^4 (Q_s t)^{-\beta}$$

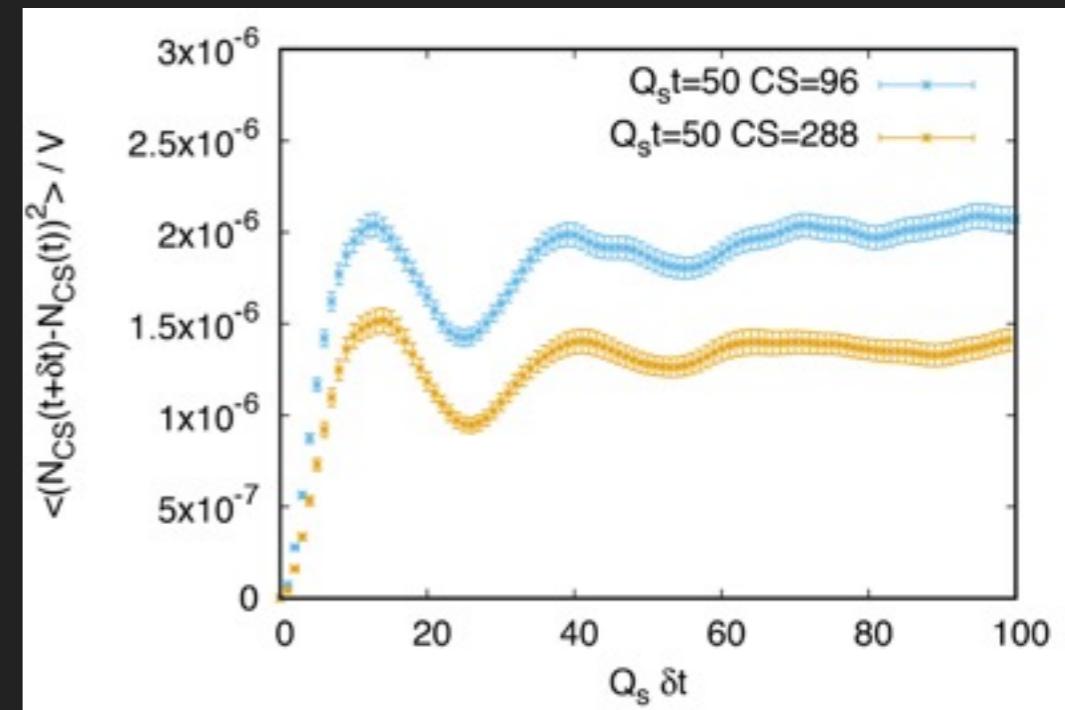
- ▶ Exponent should pick out which scales parameterize rate as we have seen separation as a function of time

CORRELATIONS

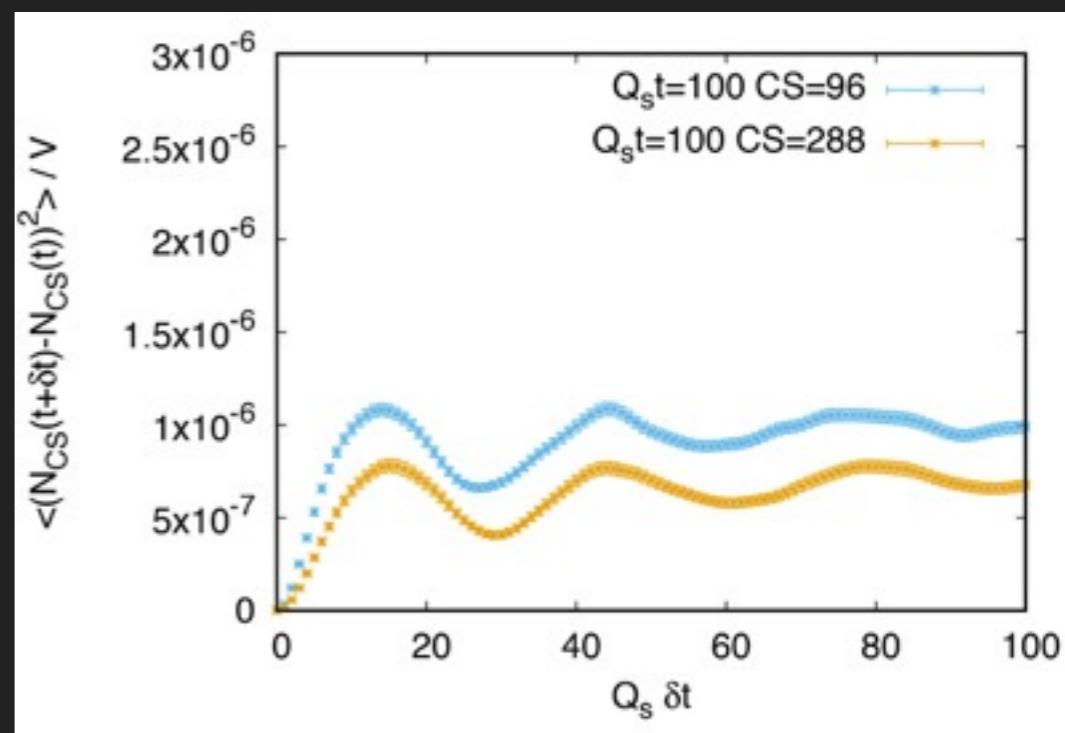
- ▶ Looking closer at correlations $(\Delta N_{CS})^2$
- ▶ Field strength fluctuations much larger in plasma
- ▶ Sensitive to cooling steps



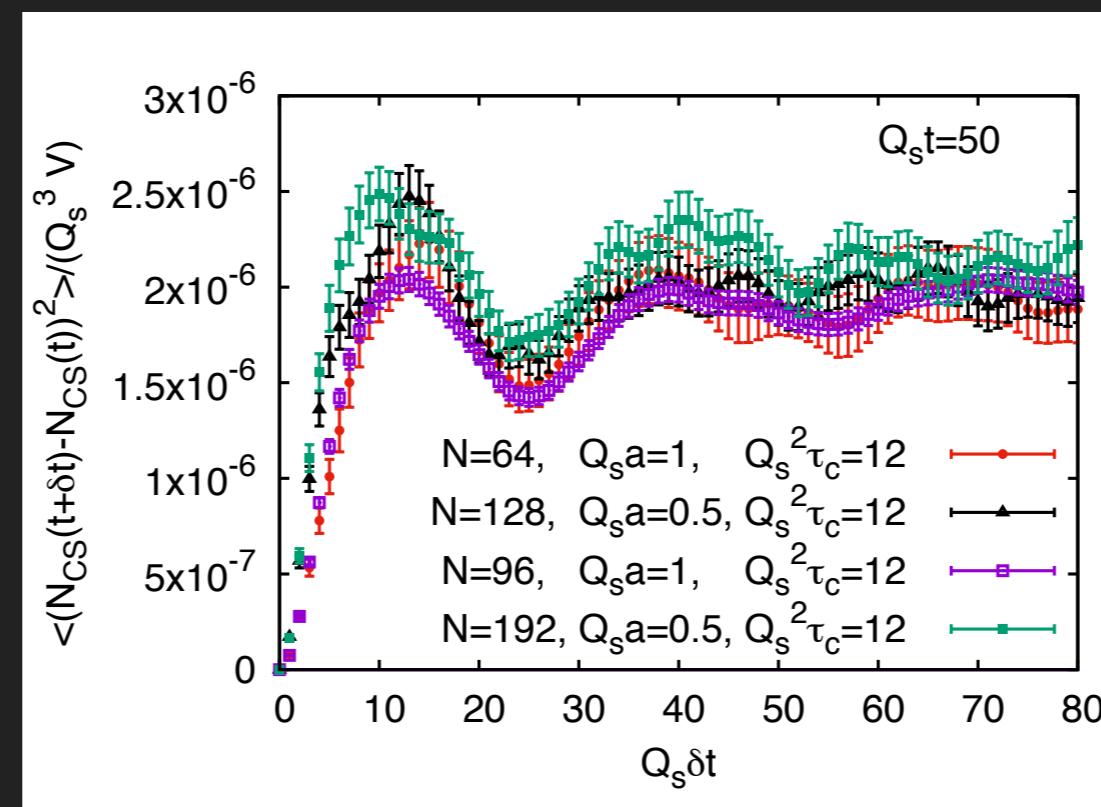
Thermal Equilibrium



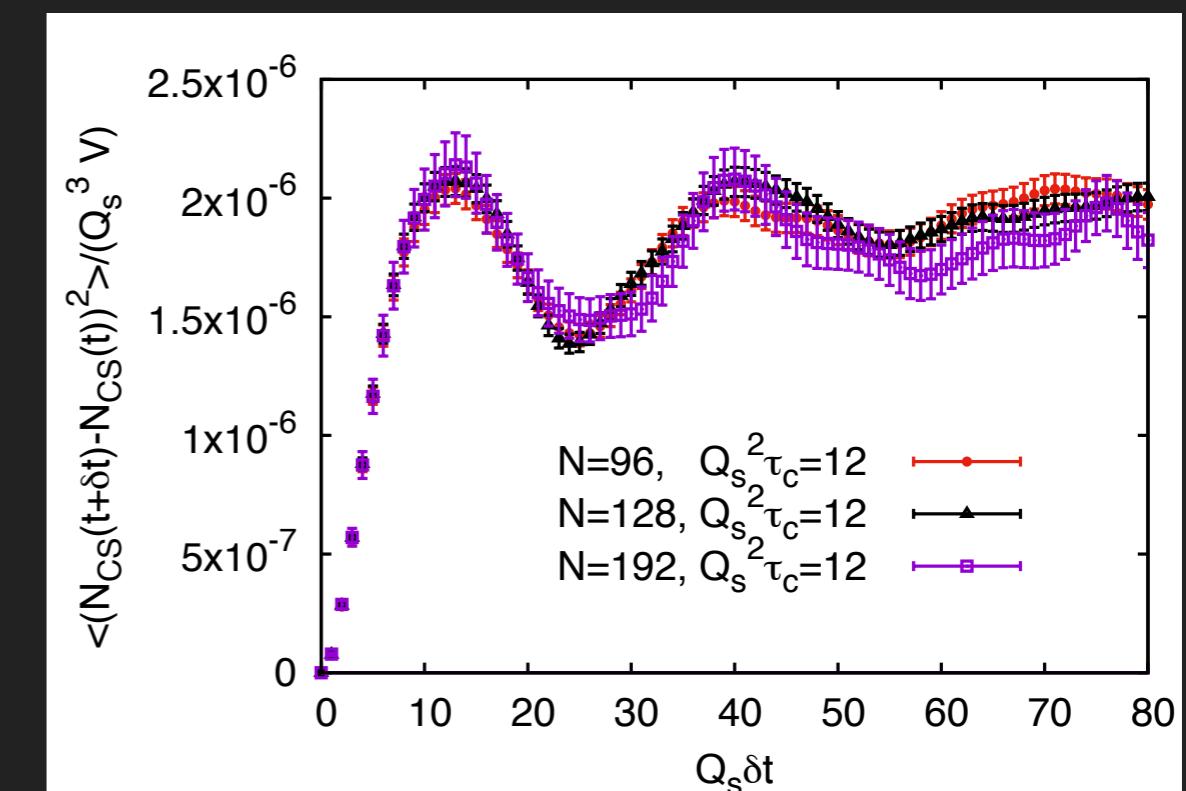
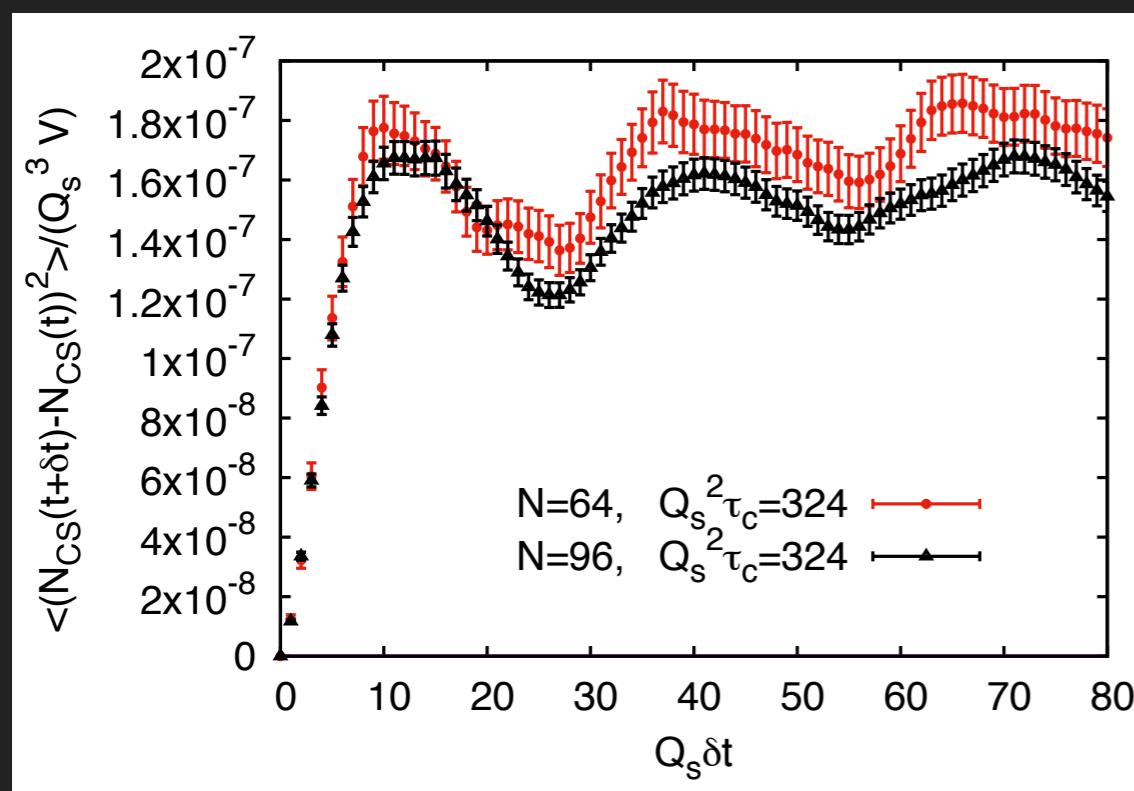
Glasma



ROBUSTNESS OF OUR RESULTS



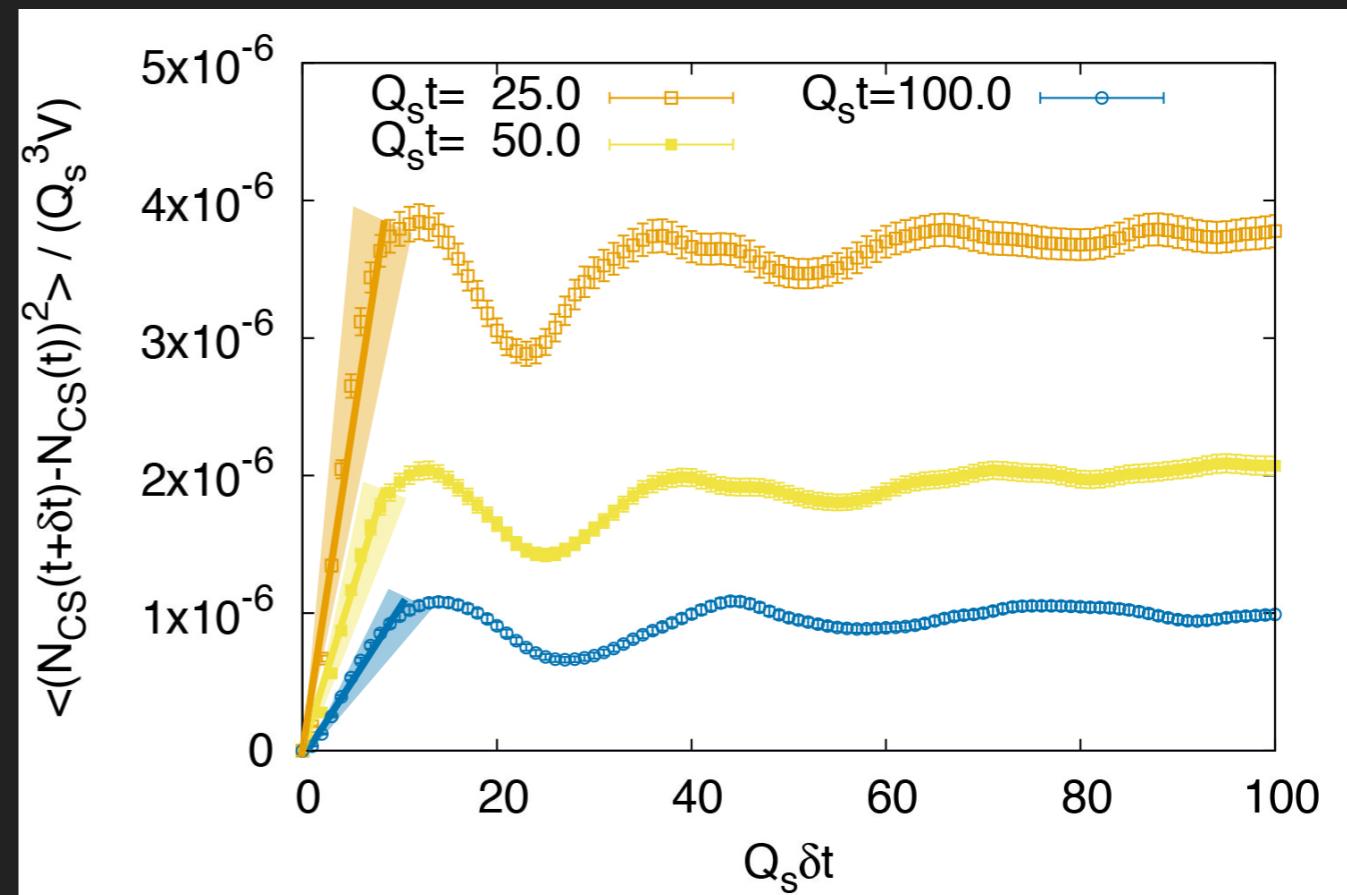
Lattice spacing
independence



Volume independence (for two cooling lengths)

RETURN OF THE SPHALERON

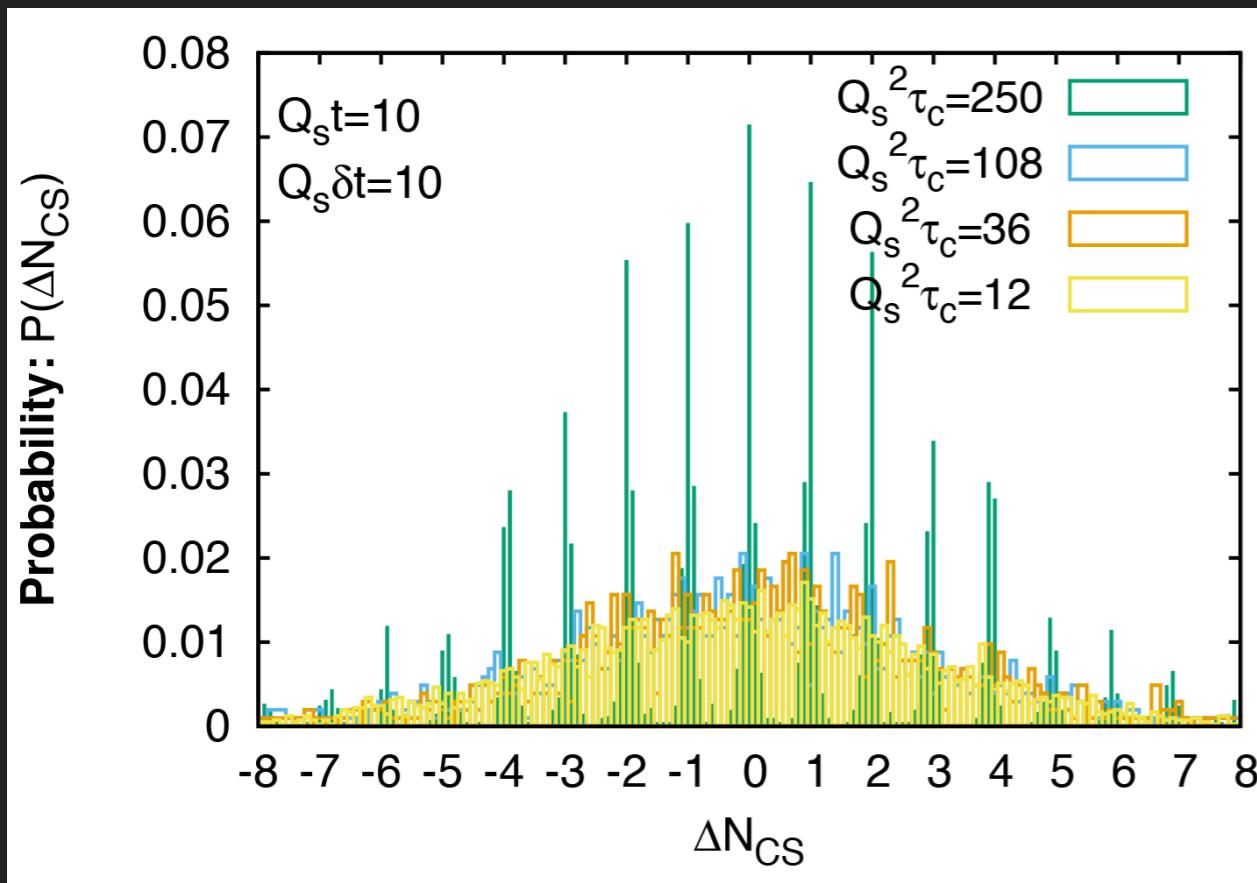
- ▶ Simple probabilistic picture
not applicable in the plasma
- ▶ Finite correlation time
- ▶ Non-Markovian features
prohibit parametrization in
terms of time dependent
sphaleron rate
- ▶ Different rate extraction than in
thermal equilibrium
- ▶ Look at initial rise in
autocorrelation function



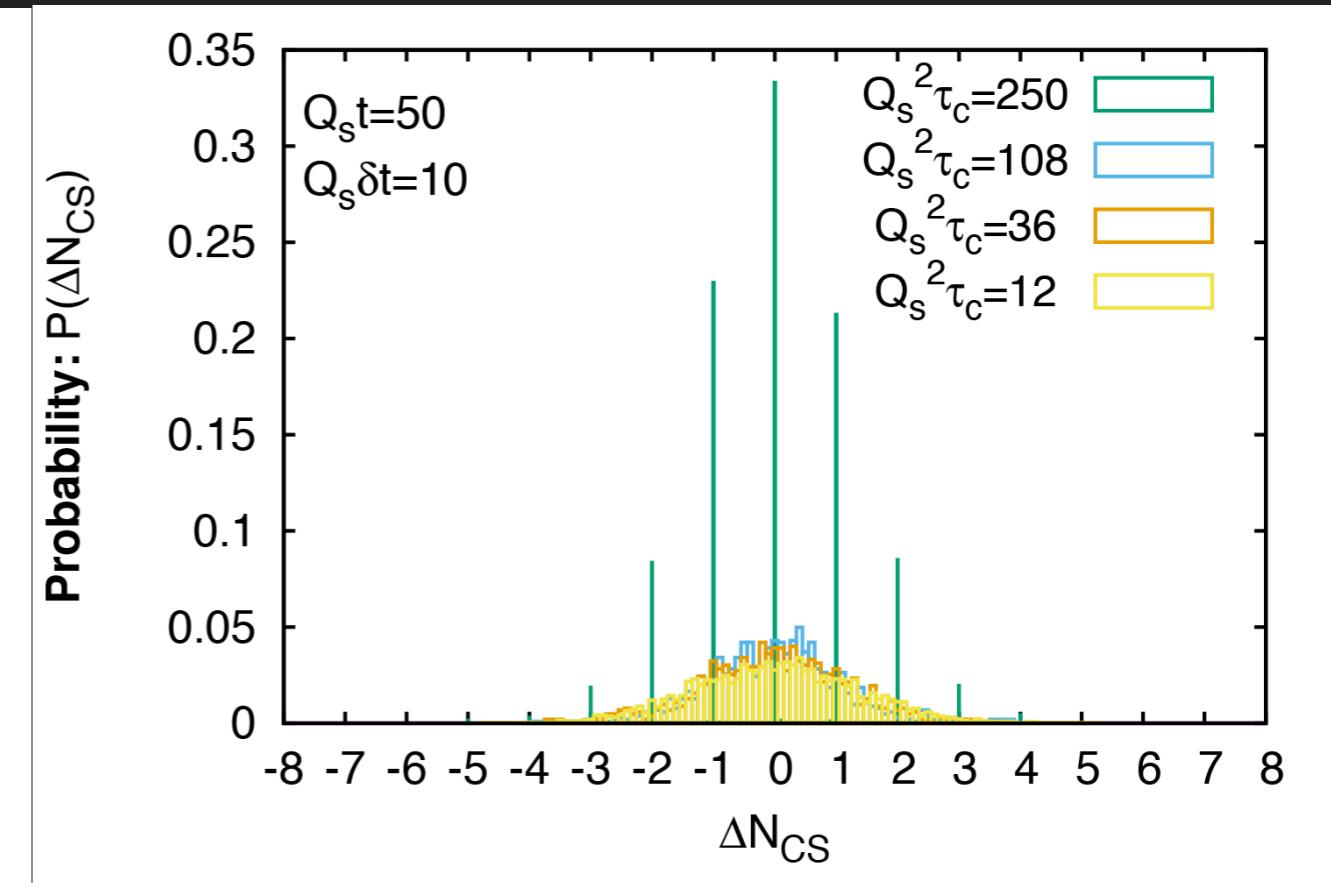
$$\Gamma_{sph}^{neq}(t) = \left\langle \frac{(N_{CS}(t + \delta t) - N_{CS}(t))^2}{V \delta t} \right\rangle_{Q_s \delta t < 10}$$

ARE THESE REALLY DELTA NCS TRANSITIONS?

- ▶ Histogram of Chern-Simons number difference for non-equilibrium



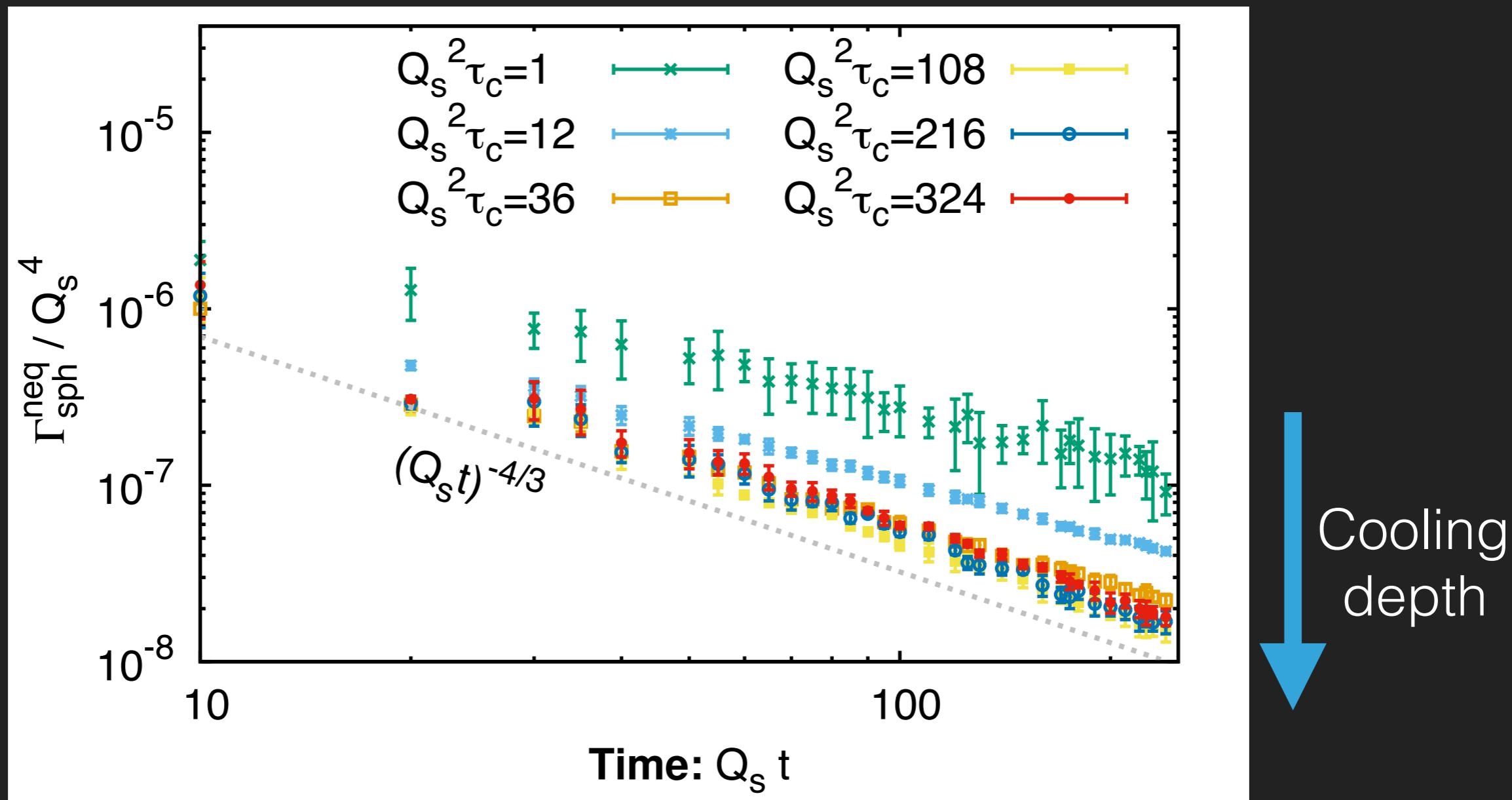
$Q_s t = 10-20$



$Q_s t = 50-60$

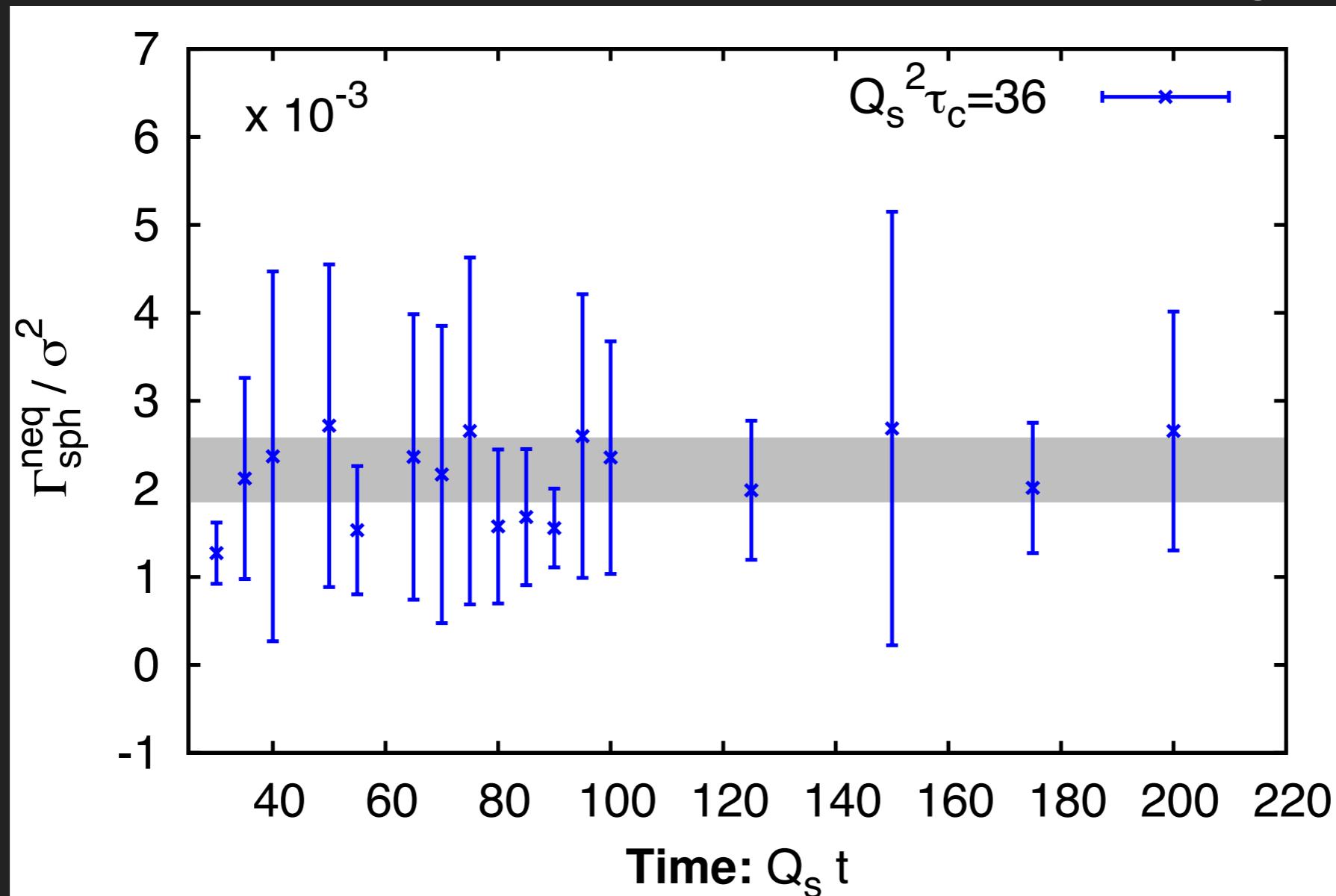
THE NON-EQUILIBRIUM SPHALERON TRANSITION RATE

- From $\Gamma_{sph}^{neq}(t) = \left\langle \frac{(N_{CS}(t + \delta t) - N_{CS}(t))^2}{V \delta t} \right\rangle_{Q_s \delta t < 10}$
- We find $\Gamma \sim Q_s^4 (Q_s t)^{-4/3}$

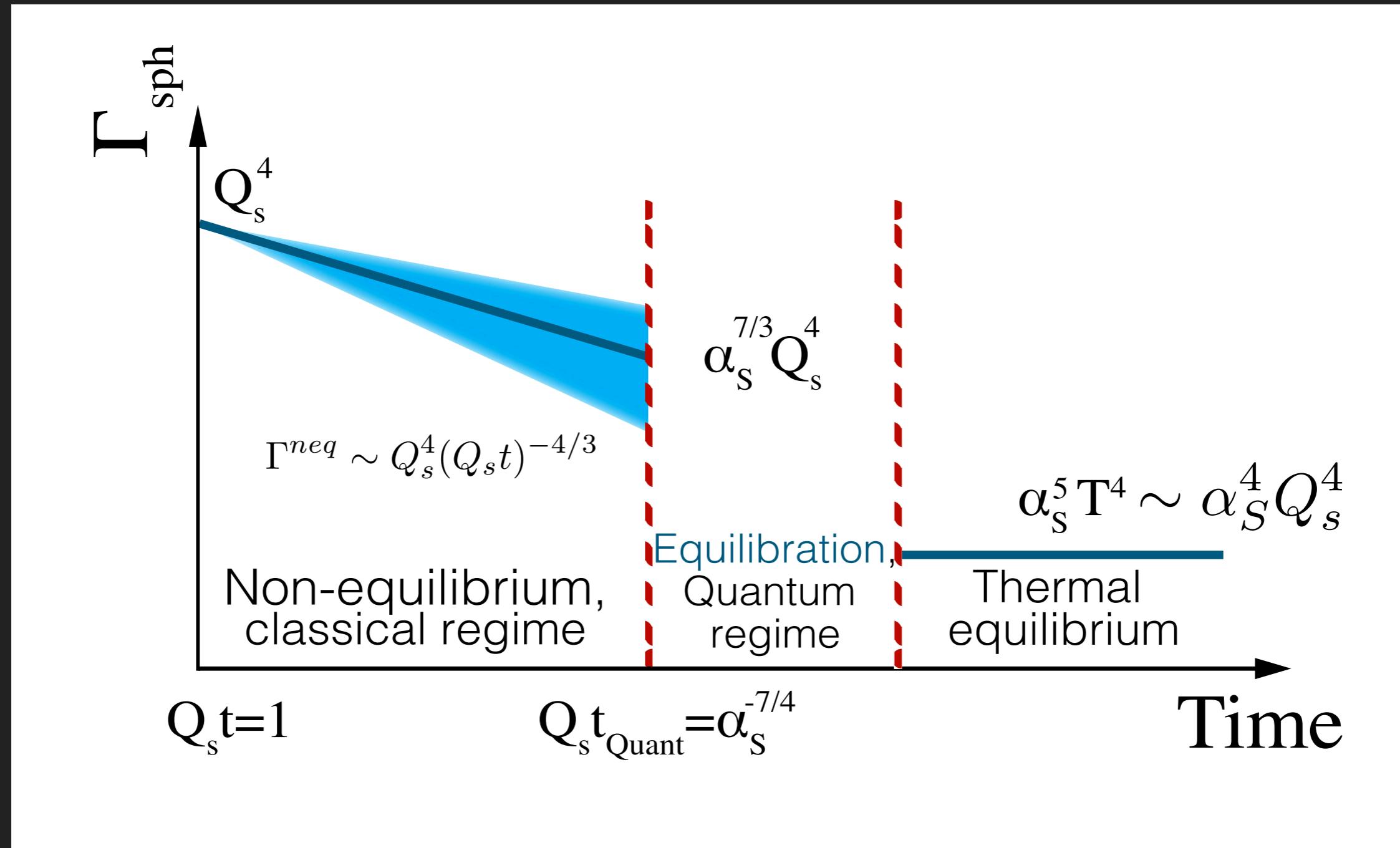


RESULTS

- ▶ In the scaling regime, we find that modes of the order of the magnetic screening scale control Γ^{neq}
- ▶ Find good agreement with $\Gamma^{neq} \sim \sigma^2 \sim l_{\text{mag}}^{-4}$



OUR PICTURE



Weak coupling picture

CONCLUSION

- ▶ Using real-time classical-statistical methods, we determine the Chern-Simons diffusion rate for a far from equilibrium over occupied non-Abelian plasma for the first time
- ▶ In the scaling regime, we find this rates scales like the magnetic length to the negative fourth power
 - ▶ Not a random walk like in thermal equilibrium
- ▶ In comparison to thermal equilibrium, off equilibrium rate is parametrically larger by $\alpha_s^{-5/3}$ at the end of classical regime
- ▶ Our studies suggest that non-equilibrium generation of topological charge should dominantly contribute to the CME

OUTLOOK

- ▶ Sphalerons in a longitudinally expanding box (more realistic plasma)
- ▶ Understand significance of field strength fluctuations vs. topological transitions in axial charge creation
- ▶ Real-time studies of CME – in progress
 - ▶ Add fermions, U(1) magnetic field
 - ▶ Studying analytical model as well
- ▶ Matching initial state dynamics to anomalous hydrodynamics
- ▶ Do phenomenology for heavy ion collisions (BES-II)

THANK YOU.